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Group Consensus of Nonlinear Hybrid Multi-Agent Systems with Self-Triggering Event Mechanism

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Abstract

The group consensus problem of nonlinear hybrid multi-agent systems (NHMAS) under self-triggering event mechanism is studied. First, a nonlinear agent dynamic model with mixed continuous and discrete states is established. Then a group consensus controller for hybrid multi-agent system (HMAS) is presented. Under the action of the controller, the agent only needs to interact with the state information of the neighbor agents in a local range, and the system can quickly realize the group consensus. Based on this definition and periodic sampling method, a self-triggering event mechanism is proposed. It can reduce the update frequency of the controller and thus reduce the energy loss by predicting the trigger time. Combined with graph theory and Lyapunov stability theory, it is proved that the proposed controller can achieve group consensus under the action of event-triggering mechanism. Finally, the correctness of the research conclusions is validated through simulation examples.

Keywords: NHMAS; Group Consensus; Self-Triggering Event Mechanism

Abbreviations

NHMAS: Nonlinear Hybrid Multi-Agent Systems; HMAS: Hybrid Multi-Agent System.

Introduction

An agent is an individual with perception, learning, and decision-making capabilities. In a specific environment, it makes decisions and takes actions based on perceived information. A single agent may face limitations when dealing with complex or large-scale tasks, leading to the concept of multi-agent systems. Currently, this concept has been applied in computer science [1], robotics [2], pattern and image recognition [3], among others. At present, research focuses on consensus control [4], cooperative control [5], swarm and formation control [6]. The group consensus evolves from the basic concept of consensus and is more complex than traditional consensus problems. To meet specific requirements, corresponding controllers are provided to enable multi-agents in the system achieving subgrouping. The positional and velocity changes of agents within each subgroup will eventually converge to consistency, while differences will exist between different subgroups.

As research delves deeper into multi-agent systems, it becomes evident that real-world applications often do not strictly adhere to either continuous or discrete systems [7,8]. Due to complex environments, situations with hybrid multi-agent systems emerge. In Xie G, et al. [9], the consensus problem of HMAS was addressed. In Lu M, et al. [10], the stability problem of second-order hybrid systems

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composed of multi-agents was solved, and by introducing new algorithms, the agents were ultimately able to achieve consensus. In Zheng Y, et al. [11], the consensus problem of hybrid systems composed of continuous and discrete agents was proposed, and stable conditions for the system were provided through research. In Zhao Q, et al. [12], the mean-square consensus problem of HMAS was studied. Based on data sampling control and considering the effects of delay and noise, conditions for mean-square consensus and an upper bound for delay were obtained. In Zheng Y, et al. [13], the bisectional consensus of HMAS composed of continuous and discrete agents was researched. By analyzing the interaction patterns of agents with different states, two effective hybrid multi-agent system consensus protocols were proposed. In Zhou Y, et al. [14], the consensus tracking problem of HMAS was studied, and a novel distributed consensus tracking control protocol was proposed. Building on the aforementioned studies, this paper investigates the consensus problem of HMAS. It considers NHMAS models and introduces the concept of hybrid group consensus, exploring the group consensus problem of NHMAS.

In the operation of multi-agent systems, individuals within the system need to continuously adjust their operational states through information exchange to accomplish their tasks. To reduce resource consumption in the system, researchers have introduced event triggered mechanisms [15]. In Wang Z, et al. [16], the problem of event- triggered consensus under collaborative control was studied, and a new event triggered mechanism for HMAS was proposed. In Li K, et al. [17], under the influence of event-triggering, the group consensus problem of HMAS was researched. A event-triggering mechanism, which does not rely on global information was introduced. To ensure that all agents can gradually achieve group consensus even in the case of discontinuous communication. Additionally, in Li B, et al. [18], a dynamic triggered mechanism was developed for the leader-following group consensus problem. This mechanism relies on auxiliary dynamic variables to effectively address the consensus issue. In Li H, et al. [19], proposed a new distributed triggered communication strategy was proposed for MAS under input saturation conditions, to achieve the group consensus under fixed topology. In Cai Y, et al. [20], for nonlinear systems affected by perturbations, a fixed triggering scheme was proposed, achieving formation tracking under reduced disturbance. Literature Chai X, et al. [21], studied the formation control problem of NHMAS, proposing an event-triggered strategy based on periodic sampling. Each agent synchronizes sampling and periodically monitors the event-triggering function. Therefore, based on the sampled control event-triggering method, this paper proposes a self-triggered event mechanism. In which can calculate and predict the next triggering time, further reducing the controller updates.

This paper makes the following innovations and contributions. We design a group consensus controller for NHMAS. Under the action of this controller, agents only need to interact with neighboring agents within a local range to exchange state information, enabling the system to achieve subgroup consensus quickly. Based on the sampled control event-triggering method, we further propose a self-triggering event mechanism. This mechanism can predict the next triggering time through calculations, thereby reducing the update frequency of the controller. Therefore, effectively lowering system energy consumption and reduce communication overhead. Since the self-triggering event mechanism samples and predicts based on a fixed cycle, it fundamentally avoids the Zeno phenomenon.

The structure of this paper is organized as follows. The second part constructs a HMAS model with nonlinear terms. And provides a definition for achieving group consensus. The third part introduces a distributed hybrid group consensus controller and the self-triggering event mechanism. Stability conditions for the system are provided, followed by stability analysis using Lyapunov stability theory. The fourth part conducts simulation examples using simulation software to validate the correctness of the conclusions.

Sign: R is a set of real numbers, R^m is a collection of real vectors. The rank transformation of matrix A is A^T. The transposition of vector p is p^T. x^{-1} represents the inverse of the matrix. $\|\cdot\|_x$ is the x-norm of the matrix.

Problem Statement

Preliminaries: The study in this paper is based on an undirected graph (denoted as G_x). Graph $G = (V, \varepsilon, A)$ is used to represent the information interaction between agents. $V = \{1, 2......, N\}$ denotes the set of nodes. $\varepsilon \subseteq \{i, j \in V \times V\}$ represents the set of edges connecting nodes, containing all edges in G. The adjacency matrix of the graph is denoted as $A = \left[a_{ij}\right] \in R^{N\times N}$. When node can receive the message transmitted by node, $a_{ij} = 0$; otherwise, $a_{ij} = 1$. The degree matrix of the graph is denoted as $D = diag\{d_1, d_2, ..., d_N\}$, $d_i = \sum_{j=1}^{N} a_{ij}$. And define L as the Laplacian matrix of the graph, $L = (l_{ij})_{N\times N}$.

Agent Dynamics Model

Consider a NHMAS consists of N agents, each with a dynamics model containing nonlinear terms. It is assumed that the M agent has continuous states while agents $M + 1$ to N have discrete states. The dynamics model of the entire NHMAS is represented as follows

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$$\begin{cases} \dot{x}_i(t) = \psi f(x_i, t) + u_i(t), i \in I_M, \\ x_i(t_{k+1}) = \psi h_1 f(x_i, t_k) + x_i(t_k) + u_i(t_k),\ t_k = c_\iota h_1, i \in I_N / I_M, \end{cases} \quad (1)$$

where $x_i \in R^n$, $u_i \in R^n$ respectively represents the position state and control input. $f(x_i, t) \in R^n$ denotes the nonlinear term. h1 represents the sampling time interval of the discrete agents.

During the research process, whether it is the coupling between two different-state agents or the influence of nonlinear terms on the system, Both pose challenges to the study of system consensus. Therefore, through our research, we provide a definition of consensus for NHMAS.

Definition 1: In a NHMAS, individuals can satisfy the following equation

$$\begin{cases} \lim_{t\to\infty} \| x_j(t) - x_i(t) \| = 0, \quad x_{\sigma j} = x_{\sigma i}, i \in I_N \\ \lim_{t_k\to\infty} \| x_j(t_k) - x_i(t_k) \| = 0, \quad x_{\sigma j} = x_{\sigma i}, i \in I_N \\ \lim_{t\to\infty, t_k\to\infty} \| x_j(t) - x_i(t_k) \| = 0, \ x_{\sigma j} = x_{\sigma i}, i \in I_N \end{cases} \quad (2)$$

Then, the state of the NHMAS is considered to achieve group consensus.

Assumption 1: Bounded nonlinear functions satisfy

$$\| f(x_i, t) - f(x_j, t) \| \le v \| x_i - x_{\sigma i} \|, \quad (3)$$

where $\upsilon > 0$, $x_{\sigma i}$ is the grouping coefficient.

Main Results

Combining the event-triggered mechanism, we present a group consensus controller for NHMAS. To ensure that the provided controller can achieve group consensus in NHMAS. The controller is as follows

$$\begin{cases} u_i(t) = \psi\left(\sum_{j\in N_t} a_{ij}\left(x_j(t_\varsigma^i) - x_i(t_\varsigma^i) \right) + \alpha \sum_{j\in N_t} l_{ij} x_{\sigma j} \right), t_\varsigma^i = c_\iota h_2, c_\iota \in N^+, i \in I_M \\ u_i(t_k) = \psi h_1\left(\sum_{j\in N_t} a_{ij}\left(x_j(t_{k\xi}^i) - x_i(t_{k\xi}^i) \right) + \alpha \sum_{j\in N_t} l_{ij} x_{\sigma j} \right), t_k \in \left[t_{k\xi}^i, t_{k\xi+1}^i \right) i \in I_N / I_M \end{cases} \quad (4)$$

Where ψ and α both represent control gains. $x_{\sigma j}$ denotes the grouping coefficient, which can achieve group consensus through different implementations of $x_{\sigma j}$. c_ι can be taken 1, 2, 3......*N*. $x(t_\varsigma^i), x(t_{k\xi}^i)$ respectively denotes the state at the latest triggering time of the corresponding agent.

The definition of joint measurement is given

$$\begin{cases} \rho_i(t) = \sum_{j\in N_i} a_{ij}\left(x_j(t) - x_i(t) \right), i \in I_M, \\ \rho_i(t_k) = \sum_{j\in N_i} a_{ij}\left(x_j(t_k) - x_i(t_k) \right), i \in I_N / I_M. \end{cases} \quad (5)$$

Based on the joint measurement method, the joint measurement error of the hybrid system is provided

$$\begin{cases} e_i(t) = \rho_i(t_\varsigma^i) - \rho_i(t), t \in \left[t_\varsigma^i, t_{\varsigma+1}^i \right), i \in I_M, \\ e_i(t_k) = \rho_i(t_{k\xi}^i) - \rho_i(t_k), t_k \in \left[t_{k\xi}^i, t_{k\xi+1}^i \right), i \in I_N / I_M. \end{cases} \quad (6)$$

In this section, an event-triggering mechanism is designed based on joint measurement errors and sampling detection methods. During the operation of the hybrid system, computations are performed according to the designed sampling instants. By computing the current state information of the agent, the next triggering instant is predicted. The obtained joint measurement error is then compared with the designed triggering threshold to determine whether the system controller needs to be updated. Furthermore, reducing the frequency of controller updates helps to save energy consumption.

The event-triggering condition is designed as follows

$$\begin{cases} \| e_i(t_s) \| \le \frac{\vartheta}{\sqrt{2+2\vartheta}} \| \rho_i(t_s) \|, t_s = c_\iota h_2, c_\iota \in N^+, i \in I_M, \\ \| e_i(t_k) \| \le \frac{\vartheta}{\sqrt{2+2\vartheta}} \| \rho_i(t_k) \|, t_k = c_\iota h_1, c_\iota \in N^+, i \in I_N / I_M, \end{cases} \quad (7)$$

In the equation, ϑ represents the control gain. Assuming the aforementioned $h_1 = h_2 = h$, it indicates that all agents in the studied HMAS have the same sampling period. Therefore, the above triggering condition can be defined as

$$\| e_i(k) \| \le \frac{\vartheta}{\sqrt{2+2\vartheta}} \| \rho_i(k) \|, k = c_\iota h, c_\iota \in N^+, i \in I_N. \quad (8)$$

To predict the next triggering instant, provide the growth rate Δe_i of the joint measurement error. Assuming that no triggering occurs at the next sampling instant k + 1, we have

$$\begin{aligned} \Delta e_i(k) &= e_i(k+1) - e_i(k) = \rho_i(k+1) - \rho_i(k) \\ &= \left\| -\sum_{j\in N_i} a_{ij}\left[\{ x_j(k+1) - x_j(k) \right] - \left[x_i(k+1) - x_i(k) \} \right] \right\| \\ &= \left\| \psi h \sum_{j\in N_i} a_{ij}\left[p_i(k_\xi^i) \rho j(k_{\xi*}^i) \right] \right\|. \end{aligned} \quad (9)$$

Where $k_{\xi*}^i$ represents the latest triggering time of neighbor of agent *i*. Observations show that $\Delta e_i(k)$ to $\Delta e_i(k_\xi^i)$ stays the same until a new agent triggers.

The relevant parameters for the self-triggering event mechanism

$$\begin{aligned} & V_b = \Delta e_i(\delta_b), Z_b = \frac{\vartheta}{\sqrt{2+2\vartheta}} \| \rho_i(k_\xi^i) \|, \delta_b = l_b \cdot h, \\ & K_b = K_{b-1} + V_{b-1}(\delta_b - \delta_{b-1}), \upsilon_b = \frac{Z_b - K_b}{V_b}, C_p = \delta_b + \upsilon_b. \end{aligned} \quad (10)$$

Where V_b represents the growth rate of the measurement error. Z_b represents the threshold of error conditions. K_b represents the specific value of the measurement error reached at the current time δ_p. δ_p is the value of the step size of the table running multiplied by the sampling interval. C_p represents the next trigger time predicted by the self-triggering event mechanism.

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Theorem 1: Consider a NHMAS (1), under the event-triggering condition (6) and influence of the group consensus controller (4), when $0 < h \le \frac{-2-2\gamma_M}{\psi^2\gamma_M\lambda_n + \gamma^2{}_M\lambda_n\psi}$ and $h \le \frac{-1}{\psi\|L\| + \psi\|L\|\gamma_M}$, the NHMAS can achieve group consensus.

Proof: Combining the model of the NHMAS (1) and the controller (4), we can obtain

$$\begin{cases} x_i(t) = x_i(t_k) + \psi(t-t_k)\left(f(x_i,t) + \sum_{j\in N_i} a_{ij}\left(x_j(t^i_{k\varsigma}) - x_i(t^i_{k\varsigma})\right) + \alpha\sum_{j\in N_i} l_{ij}x_{\sigma j}\right), t\in(k,k+1], i\in I_M, \\ x_i(t_{k+1}) = x_i(t_k) + \psi h\left(f(x_i,t) + \sum_{j\in N_i} a_{ij}\left(x_j(t^i_{k\xi}) - x_i(t^i_{k\xi})\right) + \alpha\sum_{j\in N_i} l_{ij}x_{\sigma j}\right), i\in I_N/I_M. \end{cases} \quad (11)$$

It can be concluded that triggering does not occur when the agents are at state $(k, k+1)$. Therefore, the above MAS can be uniformly written as

$$x_i(t_{k+1}) = x_i(t_k) + \psi h\left(f(x_i,t) + \sum_{j\in N_i} a_{ij}\left(x_j(t^i_{k\xi}) - x_i(t^i_{k\xi})\right) + \alpha\sum_{j\in N_i} l_{ij}x_{\sigma j}\right), i\in I_N. \quad (12)$$

Assumption

$$x(k) = [x_1(t_k), x_2(t_k), \dots, x_N(t_k)]^T,$$
$$e(k) = [e_1(t_k), e_2(t_k), \dots, e_N(t_k)]^T,$$
$$\rho(k) = [\rho_1(t_k), \rho_2(t_k), \dots, \rho_N(t_k)]^T,$$
$$f(k) = [f_1(t_k), f_2(t_k), \dots, f_N(t_k)]^T,$$
$$x_\sigma(k) = [x_{\sigma 1}(t_k), x_{\sigma 2}(t_k), \dots, x_{\sigma N}(t_k)]^T.$$

Because of $p(k^i_\xi) = p(k) + e(k)$, the system (12) can be rewritten as

$$x(k+1) = x(k) + \psi h\left(f(k) + Lx(k) + e(k) + \alpha Lx_\sigma(k)\right), i\in I_N. \quad (13)$$

Construct the following Lyapunov function

$$V(k) = \frac{1}{2}x^T(k)Lx(k). \quad (14)$$

So, further one can obtain

$$\begin{aligned} V(k+1) &= \frac{1}{2}x(k+1)^T Lx(k+1) \\ &= \frac{1}{2}(hf(x) + x(k) + hLx(k) + he(k) + \alpha hLx_\sigma(k))^T L(hf(x) \\ &\quad + x(k) + hLx(k) + he(k) + \alpha hLx_\sigma(k)) \\ &= \frac{1}{2}(x(k)^T + hf(x)^T + hx(k)^T L^T + he(k)^T + \alpha hLx_\sigma(k)^T L^T)L \\ &\quad \times(x(k) + hLx(k) + he(k) + \alpha hLx_\sigma(k)). \end{aligned} \quad (15)$$

Simplifying and rearranging the above equation (15), we get

$$\begin{aligned} V(k+1) = \frac{1}{2}(&x(k)^T Lx(k) + \psi hx(k)^T Lf(k) + \psi^2h^2f(k)^T Lf(k) + \psi^2 \\ &\times h^2x(k)^T L^2f(k) + \psi^2h^2e(k)^T Lf(k) + h^2\alpha\psi^2x_\sigma(k)^T L^T L \\ &\times f(k) + \psi^2h^2f(k)^T L^2x(k) + \psi^2h^2f(k)^T Le(k) + \alpha h^2\psi^2 \\ &\times f(k)^T L^2x_\sigma(k) + h\psi f(k)^T Lx(k) + h\psi x(k)^T L^T Lx(k) \\ &+ \psi he(k)^T Lx(k) + h\psi\alpha x_\sigma(k)^T L^T Lx(k) + \psi hx(k)^T L^2x(k) \\ &+ \psi^2h^2x(k)^T L^T L^2x(k) + \psi^2h^2e(k)^T L^2x(k) + \psi^2\alpha h^2x_\sigma(k)^T L^T \\ &\times L^2x(k) + \psi hx(k)^T Le(k) + \psi^2h^2x(k)^T L^T Le(k) + \psi^2h^2e(k)^T \\ &\times Le(k) + \psi^2\alpha h^2x_\sigma(k)^T L^T Le(k) + \alpha\psi hx(k)^T L^2x_\sigma(k) + \alpha\psi^2 \\ &\times h^2x(k)^T L^T L^2x_\sigma(k) + \alpha\psi^2h^2e(k)^T L^2x_\sigma(k) + \alpha^2h^2\psi^2x_\sigma(k)^T \\ &\times L^3x_\sigma(k)). \end{aligned} \quad (16)$$

Further, by solving the difference equation of the Lyapunov function, it is obtained that

$$\begin{aligned} \Delta V(k) = \frac{1}{2}\Big(&\psi hx(k)^T Lf(k) + \psi^2h^2f(k)^T Lf(k) + \psi^2h^2x(k)^T \\ &\times L^2f(k) + \psi^2h^2e(k)^T Lf(k) + h^2\alpha\psi^2x_\sigma(k)^T L^T Lf(k) \\ &+ \psi^2h^2f(k)^T L^2x(k) + \psi^2h^2f(k)^T Le(k) + \psi^2h^2x(k)^T \\ &\times L^2x_\sigma(k) + h\psi f(k)^T Lx(k) + h\psi x(k)^T L^T Lx(k) + \psi h \\ &\times e(k)^T Lx(k) + h\psi\alpha x_\sigma(k)^T L^T Lx(k) + \psi hx(k)^T L^2x(k) \\ &+ \psi^2h^2x(k)^T L^T L^2x(k) + \psi^2h^2e(k)^T L^2x(k) + \psi^2\alpha h^2x_\sigma(k)^T \\ &\times L^T L^2x(k) + \psi hx(k)^T Le(k) + \psi^2h^2x(k)^T L^T Le(k) + \psi^2h^2 \\ &\times e(k)^T Le(k) + \psi^2\alpha h^2x_\sigma(k)^T L^T Le(k) + \alpha\psi hx(k)^T L^2x_\sigma(k) \\ &+ \alpha\psi^2h^2x(k)^T L^T L^2x_\sigma(k) + \alpha\psi^2h^2e(k)^T L^2x_\sigma(k) + \alpha h^2\psi^2 \\ &\times x_\sigma(k)^T L^3x_\sigma(k)\Big). \end{aligned} \quad (17)$$

Let $p(k) = Lx(k)$ simplify the above equation to:

$$\begin{aligned} \Delta V(k) = \frac{1}{2}\Big(&\psi h\rho(k)^T f(k) + \psi^2h^2f(k)^T Lf(k) + \psi^2h^2p(k)^T Lf(k) + \psi^2 \\ &\times h^2e(k)^T Lf(k) + h^2\alpha\psi^2x_\sigma(k)^T L^T Lf(k) + \psi^2h^2f(k)^T Lp(k) \\ &+ \psi^2h^2f(k)^T Le(k) + \alpha h^2\psi^2f(k)^T L^2x_\sigma(k) + h\psi f(k)^T p(k) + h \\ &\times\psi p(k)^T p(k) + \psi he(k)^T p(k) + h\psi\alpha x_\sigma(k)^T L^T p(k) + \psi hp(k)^T \\ &\times p(k) + \psi^2h^2p(k)^T Lp(k) + \psi^2h^2e(k)^T Lp(k) + \psi^2\alpha h^2x_\sigma(k)^T L^T \\ &\times Lp(k) + \psi hp(k)^T e(k) + \psi^2h^2p(k)^T Le(k) + \psi^2h^2e(k)^T Le(k) \\ &+ \psi^2\alpha h^2x_\sigma(k)^T L^T Le(k) + \alpha\psi hp(k)^T Lx_\sigma(k) + \alpha\psi^2h^2p(k)^T L^2 \\ &\times x_\sigma(k) + \alpha\psi^2h^2e(k)^T L^2x_\sigma(k) + \alpha^2h^2\psi^2x_\sigma(k)^T L^3x_\sigma(k)\Big). \end{aligned} \quad (18)$$

From the inequality $\|e_i(k)\| \le \frac{\vartheta}{\sqrt{2+2\vartheta}}\|p_i(k)\|$ and $p_i(k) = -p_ik)$, we have $\|e_i(k)\| \le \gamma_M\|p_i(k)\|$, and can Obtain

$$e^T(k)e(k) \le \gamma^2{}_M p^T(k)p(k)$$
$$e^T(k)Le(k) \le \gamma^2{}_M\lambda_n p^T(k)p(k)$$
$$p^T(k)e(k) \le \gamma_M p^T(k)p(k)$$
$$p^T(k)Le(k) \le \gamma_M\lambda_n p^T(k)p(k)$$
$$e^T(k)p(k) \le \gamma_M p^T(k)p(k)$$
$$e^T(k)Lp(k) \le \gamma_M\lambda_n p^T(k)p(k)$$
$$p^T(k)Lp(k) \le \lambda_n p^T(k)p(k)$$

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Where $\gamma_M = \max\left\{\frac{\vartheta}{\sqrt{2+2\vartheta}}\right\}$ and λ_n are the largest eigenvalues of the Laplacian matrix L, so get

$$\begin{aligned}\Delta V(k) \le \frac{1}{2}\Big(&\left(2\psi h + 2\psi h\gamma_M + \psi^2 h^2\lambda_n + 2\psi^2 h^2\gamma_M\lambda_n + \psi^2 h^2 \gamma^2{}_M\lambda_n\right)\\ &\times p^T(k)p(k) + h\psi f(k)^T p(k) + h\psi\alpha x_\sigma(k)^T L^T p(k) + \psi h p(k)^T\\ &\times f(k) + \psi^2 h^2 f(k)^T Lf(k) + \psi^2 h^2 p(k)^T Lf(k) + \psi^2 h^2 e(k)^T\\ &\times Lf(k) + h^2\alpha\psi^2 x\sigma(k)^T L^T Lf(k) + \psi^2 h^2 f(k)^T Lp(k) + \psi^2\\ &\times \alpha h^2 x_\sigma(k)^T L^T Lp(k) + \psi^2 h^2 f(k)^T Le(k) + \psi^2\alpha h^2 x_\sigma(k)^T L^T\\ &\times Le(k) + \alpha\psi h p(k)^T Lx_\sigma(k) + \alpha h^2\psi^2 f(k)^T L^2 x_\sigma(k) + \alpha\psi^2 h^2\\ &\times p(k)^T L^2 x_\sigma(k) + \alpha\psi^2 h^2 e(k)^T L^2 x_\sigma(k)\Big).\end{aligned} \quad (19)$$

By using the properties of norms, we can manipulate the above equation to obtain

$$\begin{aligned}\Delta V(k) \le \frac{1}{2}\Big(&\left(2\psi h + 2\psi h\gamma_M + \psi^2 h^2\lambda_n + 2\psi^2 h^2\gamma_M\lambda_n + \psi^2 h^2\gamma^2{}_M\lambda_n\right)\\ &\times\left\|p^T(k)p(k)\right\| + \left(\psi h + \psi^2 h^2\|L\| + \psi^2 h^2\|L\|\gamma_M\right)\left\|f(k)^T\right\|\|p(k)\|\\ &+\left(\psi h + \psi^2 h^2\|L\| + \psi^2 h^2\|L\|\gamma_M\right)\|f(k)\|\left\|p(k)^T\right\| + h\psi\alpha\left\|x_\sigma(k)^T\right\|\\ &\times\left\|L^T\right\|\|p(k)\| + \alpha\psi^2 h^2\left\|f(k)^T\right\|\|L\|\|f(k)\| + h^2\alpha\psi^2\left\|x_\sigma(k)^T\right\|\left\|L^2\right\|\\ &\times\|f(k)\| + \psi^2 ah^2\left\|x_\sigma(k)^T\right\|\left\|L^2\right\|\|p(k)\| + \psi^2\alpha h^2\left\|x_\sigma(k)^T\right\|\left\|L^2\right\|\\ &\times\|e(k)\| + \alpha\psi h\left\|p(k)^T\right\|\|L\|\|x_\sigma(K)\| + \alpha h^2\psi^2\left\|f(k)^T\right\|\left\|L^2\right\|\|x_\sigma(k)\|\\ &+\alpha\psi^2 h^2\left\|p(k)^T\right\|\left\|L^2\right\|\|x_\sigma(k)\| + \alpha\psi^2 h^2\left\|e(k)^T\right\|\left\|L^2\right\|\|x_\sigma(k)\|\Big).\end{aligned} \quad (20)$$

Splitting equation (20) into $\Delta V(k) \le \Delta V_1(k) + \Delta V_2(k)$, where

$$\begin{aligned}\Delta V_1(k) = \frac{1}{2}\Big(&\left(2\psi h + 2\psi h\gamma_M + \psi^2 h^2\lambda_n + 2\psi^2 h^2\gamma_M\lambda_n + \psi^2 h^2\gamma^2{}_M\lambda_n\right)\\ &\times\left\|p^T(k)p(k)\right\| + \left(\psi h + \psi^2 h^2\|L\| + \psi^2 h^2\|L\|\gamma_M\right)\left\|f(k)^T\right\|\\ &\times\|p(k)\| + \left(\psi h + \psi^2 h^2\|L\| + \psi^2 h^2\|L\|\gamma_M\right)\|f(k)\|\left\|p(k)^T\right\|\Big).\end{aligned} \quad (21)$$

By computation, it can be obtained that when $0 < h \le \frac{-2-2\gamma_M}{\psi\left(2\gamma_M\lambda_n + \lambda_n + \gamma^2{}_M\lambda_n\right)}$ and $h \le \frac{-1}{\psi\left(\|L\| + \|L\|\gamma_M\right)}$, then $\Delta V_1 \le 0$.

$$\begin{aligned}\Delta V_2(k) = \frac{1}{2}\Big(&h\psi\alpha\left\|x_\sigma(k)^T\right\|\times\left\|L^T\right\|\|p(k)\| + \alpha\psi^2 h^2\left\|f(k)^T\right\|\|L\|\|f(k)\|\\ &+h^2\alpha\psi^2\left\|x_\sigma(k^T)\right\|\left\|L^2\right\|\|f(k)\| + \psi^2\alpha h^2\left\|x_\sigma(k)^T\right\|\left\|L^2\right\|\|p(k)\|\\ &+\psi^2 ah^2\left\|x_\sigma(k)^T\right\|\left\|L^2\right\|\|e(k)\| + \alpha\psi h\left\|p(k)^T\right\|\|L\|\|x_\sigma(k)\|\\ &+\alpha h^2\psi^2\left\|f(k)^T\right\|\left\|L^2\right\|\|x_\sigma(k)\| + \alpha\psi^2 h^2\left\|p(k)^T\right\|\left\|L^2\right\|\|x_\sigma(k)\|\\ &+\alpha\psi^2 h^2\left\|e(k)^T\right\|\left\|L^2\right\|\|x_\sigma(k)\|\Big).\end{aligned} \quad (22)$$

When $\alpha \le 0$, then $\Delta V_2 \le 0$ In summary, under assumption 1, when $0 < h \le \frac{-2-2\gamma_M}{\psi^2\gamma_M\lambda_n + \psi\lambda_n + \gamma^2{}_M\lambda_n\psi}$ and $h < \frac{-1}{\psi\gamma\|L\| + \psi\|L\| + \gamma_M}, \alpha \le 0,$ then $\Delta V \le 0$. Through the Lyapunov stability criterion of the discrete system, we deduce that the system satisfies the stability condition.

Through analysis, it is obtained that when $\left\|p_i(k)\right\| = 0$ then $\Delta V(k) = 0$. When $\left\|p_i(k)\right\| = 0$, due to $p_i(k) = -p_i k)$, it follows that $p_i(k) = 0$, and subsequently $\left\|e_i(k)\right\| = 0$. Then, through equation (5), it is found that $p_i(t_k), p_i(t)$ equals 0. Combined with the provided definition of joint measurement, it is inferred that the NHMAS studied in this chapter achieves group consensus.

The event-triggering condition designed in this chapter samples with a time interval of h, ensuring a lower bound on the time interval between any two triggering instants. This fundamentally avoids the occurrence of Zeno phenomenon.

Numerical Simulation

Consider a NHMAS composed of 6 agents, where agents 1, 2, and 3 represent continuous agents, and agents 4, 5, and 6 represent discrete individuals. The communication network is shown in Figure 1.

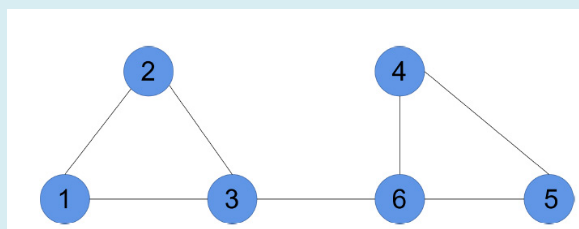

Figure 1: Structure topology of hybrid multi-agent system.

From the above figure, the Laplacian matrix L of the system can be obtained

$$L = \begin{bmatrix} 2 & -1 & -1 & 0 & 0 & 0 \\ -1 & 2 & -1 & 0 & 0 & 0 \\ -1 & -1 & 3 & 0 & 0 & -1 \\ 0 & 0 & 0 & 2 & -1 & -1 \\ 0 & 0 & 0 & -1 & 2 & -1 \\ 0 & 0 & -1 & -1 & -1 & 3 \end{bmatrix}.$$

The sampling cycle time h = 0.2s, α = −1 is selected according to the proof in the above section. The nonlinear term $f(x_i,t) = \sin x(t), x(0) = [-7\ -4\ 0\ 5\ 8\ 12]^T$ is selected as the initial position state of 6 multi- agents.

The final simulation results are as follows

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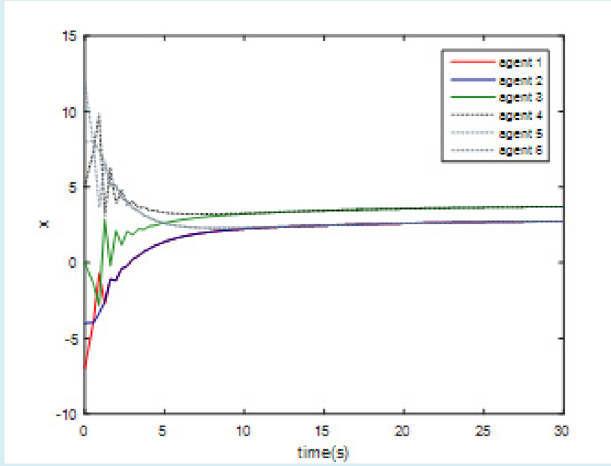

Figure 2: Time trajectory of agent states.

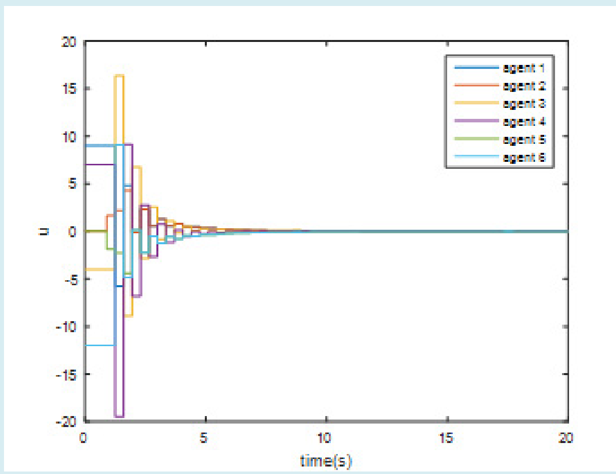

Figure 3: Control input of each agent.

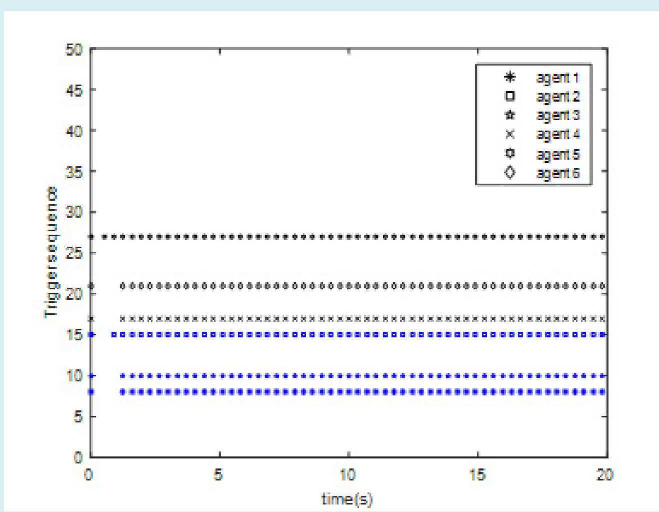

Figure 4: Triggering moment of agents under event-triggering condition.

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Simulation Result Analysis. Figure 2 intuitively demonstrates the dynamic evolution of the positions of multi- agents with different states in the hybrid multi-agent system over time. Under the regulation of the group consensus controller, the six agents successfully achieve clustering into subgroups. Where agents 1 and 2 form independent subgroups. As time progresses, the position trajectories within each subgroup gradually converge, reaching a consistent state. Figure 3 reveals the regular variation of control inputs over time for each agent. Under the action of the self-triggering event mechanism, the control inputs exhibit a step-like change pattern. Indicating that control inputs are updated only when the system satisfies specific event-triggering conditions. Otherwise, they remain unchanged from the previous triggering instant until new triggering conditions are met. Figure 4 visually displays the specific triggering time sequence for each agent under the self-triggering event mechanism. It can be observed from the figure that the triggering times of agents exhibit a discrete distribution, further demonstrating the effectiveness of the event-triggering strategy in optimizing resource utilization and reducing communication burden. In summary, the NHMAS achieves stable operation and attains group consensus under the combined action of the self-triggering event mechanism.

Conclusion

The article had explored the group consensus problem of NHMAS based on the self-triggered event mechanism. It had first presented the dynamic model of HMAS with nonlinear terms and defined group consensus. Then, a distributed hybrid agent group consensus controller had been proposed. To effectively reduce the inter-action between agents and the controller update, a self-triggered event mechanism has been introduced. This method had been able to compute the next triggering time in advance. Then further reducing the controller update and fundamentally avoiding the Zeno phenomenon. Through Lyapunov theory, it had been proven that the controller could achieve subgroup consensus under the action of the self-triggered event mechanism. Finally, simulation examples had been used to further verify the feasibility of the theoretical results.

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